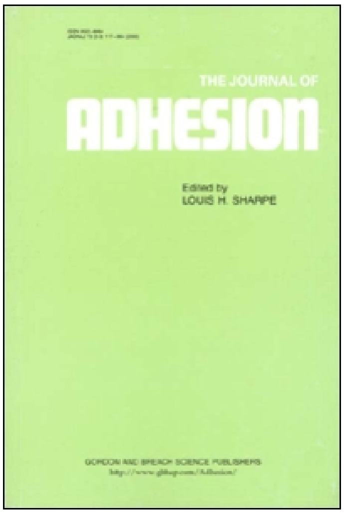

The Journal of Adhesion
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453635

Interface Stress Response of Laminated Plates Subjected to Static and Impact Loads

Toshiyuki Sawa[a]; Yuko Senoo[a]; Hirohisa Okuno[a]; Tatsumi Hagiwara[b]
[a] Yamanashi University, Kofu Yamanashi, Japan [b] Kawasaki Heavy Industry, Gifu, Japan

To cite this Article Sawa, Toshiyuki , Senoo, Yuko , Okuno, Hirohisa and Hagiwara, Tatsumi(1996) 'Interface Stress Response of Laminated Plates Subjected to Static and Impact Loads', The Journal of Adhesion, 59: 1, 1 — 16
To link to this Article: DOI: 10.1080/00218469608011072
URL: http://dx.doi.org/10.1080/00218469608011072

J. Adhesion, 1996, Vol. 59, pp. 1–16
Reprints available directly from the publisher
Photocopying permitted by license only

Interface Stress Response of Laminated Plates Subjected to Static and Impact Loads*

TOSHIYUKI SAWA**, YUKO SENOO, HIROHISA OKUNO

Yamanashi University, Takeda, Kofu, Yamanashi, 400 Japan

and

TATSUMI HAGIWARA

Kawasaki Heavy Industry, Kagamihara, Gifu, 504 Japan

(In final form January 20, 1996)

This paper deals with the stress analysis of laminated sandwich beams subjected to static loads and impact loads. When the laminated sandwich beams are subjected to static loads, stress distribution at the interfaces is analyzed, by using two-dimensional theory of elasticity, as a contact problem. When the laminated sandwich beams are subjected to impact loads, the interface stress response is analyzed using FEM (DYNA3D). Experiments were conducted. A fairly good agreement is seen between the analytical and the experimental results. The effects of the ratios of Young's moduli for each beam on the interface stress response are clarified.

KEY WORDS: Stress analysis; interfacial stress distribution; three-layered laminated sandwich beams; five-body contact problem; two-dimensional elasticity theory; finite element analysis; comparison of numerical and experimental results; effect of Young's modulus ratio.

1. INTRODUCTION

Laminated composite structures have been widely used in aerospace industry, mechanical engineering and so on, in order to lighten their weight and to increase the stiffness. In addition, laminated sandwich beams and plates have also been used in order to decrease vibrations and to increase the stiffness. Many investigations have been carried out on laminated composite structures. However, some researches have been performed on the stress analysis and the strength evaluation of laminated sandwich structures subjected to static loads. Moreover, few investigations have been carried out on the stress analysis of laminated sandwich beams and plates subjected to impact loads. In optimal designing of laminated sandwich beams and

*Presented at the International Adhesion Symposium, *IAS'94 Japan*, at the 30th Anniversary Meeting of the Adhesion Society of Japan, Yokohama, Japan November 6–10, 1994.

**Corresponding author.

plates, it is necessary to examine the stress distribution at the interface of such beams and plates subjected to static and impact loads.

In this paper, the stress distribution and the deformation of laminated sandwich beams are analyzed as a five-layered contact problem by using a two-dimensional theory of elasticity. In the analysis, each layer is replaced by a finite strip. The beams are supported at both ends and subjected to static loads at the center. The effects of the thickness of the face and core materials and the ratios of Young's moduli among the core and the face materials on the stress distributions at the interfaces and the deformations of the beams are clarified by the numerical calculations. In addition, the interface stress responses of the beams are analyzed using the finite element method (DYNA3D). The effects of the ratios among the Young's moduli of the beams on the interface stress response and the impact energy are examined. For verification, experiments were performed on the deformation of the beams subjected to static loads. Strain response of the laminated sandwich beams subjected to impact loads was measured by strain gauges. The analytical results are compared with the experimental ones.

2. ANALYSIS

2.1. Analysis of Laminated Sandwich Beams Subjected to Static Loads

Figure 1 shows a simply supported, five-layered, laminated sandwich beam subjected to a static load. Figure 2 shows a model for analysis. In order to analyze the stress distribution and the deformation of laminated sandwich beam, each layer is replaced by finite strip and denoted as layer [I], [II], [III], [IV] and [V], respectively, as shown in Figure 2. Their Young's modulus is denoted as E_i, Poisson's ratio as ν_i and the thickness as $2h_i (i = 1, 2, \ldots, 5)$, respectively. The length of each finite strip is denoted by 2l. The compression, $F(x)$, is assumed to apply to the upper surface of the laminated sandwich beam within the region $x < c_1$. At the lower surface of finite strip [V], the uniform force, $G(x)$, is applied as shown in Figure 2. The interface

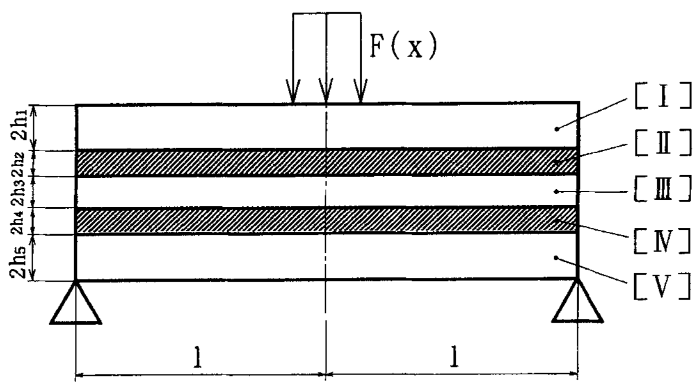

FIGURE 1 A simply supported, five-layered, laminated sandwich beam subjected to a static load.

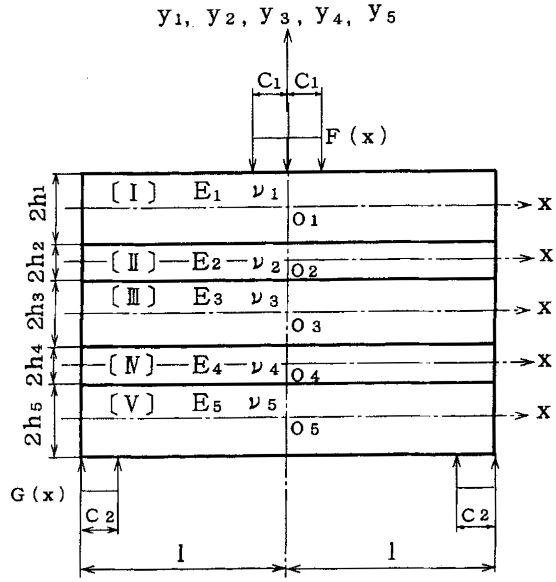

FIGURE 2 A model for analysis.

stress distributions are analyzed, by using a two-dimensional theory of elasticity, as five-body contact problem.

In the analysis, the force distributions $F(x)$ and $G(x)$ are expanded into Fourier series. The boundary conditions are expressed as the following equations, where the displacement in the x direction is denoted as u and the displacement in the y direction as v.

For finite strip [I]

$$x = \pm l : \sigma_x^{\mathrm{I}} = \tau_{xy}^{\mathrm{I}} = 0$$

$$y_1 = h_1 : \sigma_y^{\mathrm{I}} = a_0 + \sum_{s=1}^{\infty} a_s \cos\left(\frac{s\pi x}{l}\right)$$

$$\tau_{xy}^{\mathrm{I}} = 0$$

For finite strip [II]

$$x = \pm l : \sigma_x^{\mathrm{II}} = \tau_x^{\mathrm{II}} = 0$$

For finite strip [III]

$$x = \pm l : \sigma_x^{\mathrm{III}} = \tau_{xy}^{\mathrm{III}} = 0$$

For finite strip [IV]

$$x = \pm l : \sigma_x^{\mathrm{IV}} = \tau_x^{\mathrm{IV}} = 0$$

For finite strip [V]

$$x = \pm l : \sigma_x^{\mathrm{V}} = \tau_{xy}^{\mathrm{V}} = 0$$

$$y_5 = -h_5 : \sigma_y^V = b_0 + \sum_{s=1}^{\infty} b_s \cos\left(\frac{s\pi x}{l}\right)$$

$$\tau_{xy}^V = 0$$

at the interface between finite strip [I] and [II]

$$(\sigma_y^{\mathrm{I}})_{y_1=-h_1} = (\sigma_y^{\mathrm{II}})_{y_2=h_2}$$

$$(\tau_{xy}^{\mathrm{I}})_{y_1=-h_1} = (\tau_{xy}^{\mathrm{II}})_{y_2=h_2}$$

$$(u^{\mathrm{I}})_{y_1=-h_1} = (u^{\mathrm{II}})_{y_2=h_2}$$

$$\left(\frac{\partial v^{\mathrm{I}}}{\partial x}\right)_{y_1=-h_1} = \left(\frac{\partial v^{\mathrm{II}}}{\partial x}\right)_{y_2=h_2}$$

where

$$a_0 = \frac{1}{2l}\int_{-l}^{l} F(x)dx$$

$$a_s = \frac{1}{l}\int_{-l}^{l} F(x)\cos\left(\frac{s\pi x}{l}\right)dx$$

$$b_s = \frac{1}{2l}\int_{-l}^{l} G(x)dx$$

$$b_s = \frac{1}{l}\int_{-l}^{l} G(x)\cos\left(\frac{s\pi x}{l}\right)dx$$

$$(a_0 = b_0)$$

The boundary conditions at the other interfaces are expressed in the same way. Each finite strip is analyzed under the above boundary conditions using Airy's stress functions. Each stress is expressed by Eq. (1) and each displacement by Eq. (2).

$$\sigma_x = \frac{\partial^2 x}{\partial y^2}, \quad \sigma_y = \frac{\partial^2 \chi}{\partial x^2}, \quad \tau_{xy} = \frac{\partial^2 \chi}{\partial x \partial y} \tag{1}$$

$$2Gu = -\frac{\partial \chi}{\partial x} + \frac{1}{1+v}\cdot\frac{\partial \phi}{\partial y}, \quad 2Gv = -\frac{\partial \chi}{\partial y} + \frac{1}{1+v}\cdot\frac{\partial \phi}{\partial x} \tag{2}$$

where

$$\nabla^2\nabla^2\chi = 0, \quad \nabla^2\phi = 0$$

$$\left(\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$

ϕ is obtained from Eq. (3).

$$\frac{\partial^2 \phi}{\partial x \partial y} = \nabla^2 \chi \tag{3}$$

where G is shear modulus and v is Poisson's ratio. Airy's stress functions

$$\chi^{\mathrm{I}} = \chi_0^{\mathrm{I}} + \chi_1^{\mathrm{I}} + \chi_2^{\mathrm{I}} + \chi_3^{\mathrm{I}} + \chi_4^{\mathrm{I}}$$

for analyzing finite strip [I] are expressed by the following Eq. (4).

$$\chi_0^{\mathrm{I}} = \chi_0(A_0^{\mathrm{I}}, B_0^{\mathrm{I}}, x_1, y_1) = \frac{A_0^{\mathrm{I}}}{2} x_1^2 + \frac{B_0^{\mathrm{I}}}{2} y_1^2$$

$$\begin{aligned}
\chi_1^{\mathrm{I}} &= \chi_1(A_{n1}^{\mathrm{I}}, B_{s1}^{\mathrm{I}}, l_1, h_1, \alpha_n^{\mathrm{I}}, \lambda_s^{\mathrm{I}}, \Delta_{n1}^{\mathrm{I}}, \Omega_{s1}^{\mathrm{I}}, x_1, y_1) \\
&= \sum_{n=1}^{\infty} \frac{A_{n1}^{\mathrm{I}}}{\Delta_{n1}^{\mathrm{I}} \alpha_n^{\mathrm{I}^2}} [\{\alpha_n^{\mathrm{I}} l_1 \cos\mathrm{h}(\alpha_n^{\mathrm{I}} l_1) + \sin\mathrm{h}(\alpha_n^{\mathrm{I}} l_1)\} \cos\mathrm{h}(\alpha_n^{\mathrm{I}} x_1) \\
&\quad - \alpha_n^{\mathrm{I}} x_1 \sin\mathrm{h}(\alpha_n^{\mathrm{I}} l_1) \sin\mathrm{h}(\alpha_n^{\mathrm{I}} x_1)] \cos(\alpha_n^{\mathrm{I}} y_1) \\
&\quad + \sum_{s=1}^{\infty} \frac{B_{s1}^{\mathrm{I}}}{\Omega_{s1}^{\mathrm{I}} \lambda_s^{\mathrm{I}^2}} [\{\lambda_s^{\mathrm{I}} h_1 \cos\mathrm{h}(\lambda_s^{\mathrm{I}} h_1) + \sin\mathrm{h}(\lambda_s^{\mathrm{I}} h_1)\} \cos\mathrm{h}(\lambda_s^{\mathrm{I}} y_1) \\
&\quad - \lambda_s^{\mathrm{I}} y_1 \sin\mathrm{h}(\lambda_s^{\mathrm{I}} h_1) \sin\mathrm{h}(\lambda_s^{\mathrm{I}} y_1)] \cos(\lambda_s^{\mathrm{I}} x_1)
\end{aligned}$$

$$\begin{aligned}
\chi_2^{\mathrm{I}} &= \chi_2(A_{n2}^{\mathrm{I}}, B_{s2}^{\mathrm{I}}, l_1, h_1, \alpha_n^{\mathrm{I}'}, \lambda_s^{\mathrm{I}}, \Delta_{n2}^{\mathrm{I}}, \Omega_{s2}^{\mathrm{I}}, x_1, y_1) \\
&= \sum_{n=1}^{\infty} \frac{A_{n2}^{\mathrm{I}}}{\Delta_{n2}^{\mathrm{I}} \alpha_n^{\mathrm{I}'^2}} [\{\alpha_n^{\mathrm{I}'} l_1 \cos\mathrm{h}(\alpha_n^{\mathrm{I}'} l_1) + \sin\mathrm{h}(\alpha_n^{\mathrm{I}'} l_1)\} \cos\mathrm{h}(\alpha_n^{\mathrm{I}'} x_1) \\
&\quad - \alpha_n^{\mathrm{I}'} x_1 \sin\mathrm{h}(\alpha_n^{\mathrm{I}'} l_1) \sin\mathrm{h}(\alpha_n^{\mathrm{I}'} x_1)] \cos(\alpha_n^{\mathrm{I}'} y_1) \\
&\quad + \sum_{s=1}^{\infty} \frac{B_{s2}^{\mathrm{I}}}{\Omega_{s2}^{\mathrm{I}} \lambda_s^{\mathrm{I}^2}} [\{\lambda_s^{\mathrm{I}} h_1 \sin\mathrm{h}(\lambda_s^{\mathrm{I}} h_1) + \cos\mathrm{h}(\lambda_s^{\mathrm{I}} h_1)\} \sin\mathrm{h}(\lambda_s^{\mathrm{I}} y_1) \\
&\quad - \lambda_s^{\mathrm{I}} y_1 \cos\mathrm{h}(\lambda_s^{\mathrm{I}} h_1) \cos\mathrm{h}(\lambda_s^{\mathrm{I}} y_1)] \cos(\lambda_s^{\mathrm{I}} x_1)
\end{aligned}$$

$$\begin{aligned}
\chi_3^{\mathrm{I}} &= \chi_3(A_{n3}^{\mathrm{I}}, B_{s3}^{\mathrm{I}}, l_1, h_1, \alpha_n^{\mathrm{I}'}, \lambda_s^{\mathrm{I}'}, \Delta_{n3}^{\mathrm{I}}, \Omega_{s3}^{\mathrm{I}}, x_1, y_1) \\
&= -\sum_{n=1}^{\infty} \frac{A_{n3}^{\mathrm{I}}}{\Delta_{n3}^{\mathrm{I}} \alpha_n^{\mathrm{I}'^2}} [\alpha_n^{\mathrm{I}'} l_1 \sin\mathrm{h}(\alpha_n^{\mathrm{I}'} l_1) \cos\mathrm{h}(\alpha_n^{\mathrm{I}'} x_1) \\
&\quad - \alpha_n^{\mathrm{I}'} x_1 \cos\mathrm{h}(\alpha_n^{\mathrm{I}'} l_1) \sin\mathrm{h}(\alpha_n^{\mathrm{I}'} x_1)] \cos(\alpha_n^{\mathrm{I}'} y_1) \\
&\quad - \sum_{s=1}^{\infty} \frac{B_{s3}^{\mathrm{I}}}{\Omega_{s3}^{\mathrm{I}} \lambda_s^{\mathrm{I}'^2}} [\lambda_s^{\mathrm{I}'} h_1 \sin\mathrm{h}(\lambda_s^{\mathrm{I}'} h_1) \cos\mathrm{h}(\lambda_s^{\mathrm{I}'} y_1) \\
&\quad - \lambda_s^{\mathrm{I}'} y_1 \cos\mathrm{h}(\lambda_s^{\mathrm{I}'} h_1)\} \sin\mathrm{h}(\lambda_s^{\mathrm{I}'} y_1)] \cos(\lambda_s^{\mathrm{I}'} x_1)
\end{aligned}$$

$$\begin{aligned}
\chi_4^{\mathrm{I}} &= \chi_4(A_{n4}^{\mathrm{I}}, B_{s4}^{\mathrm{I}}, l_1, h_1, \alpha_n^{\mathrm{I}}, \lambda_s^{\mathrm{I}'}, \Delta_{n4}^{\mathrm{I}}, \Omega_{s4}^{\mathrm{I}}, x_1, y_1) \\
&= -\sum_{n=1}^{\infty} \frac{A_{n4}^{\mathrm{I}}}{\Delta_{n4}^{\mathrm{I}} \alpha_n^{\mathrm{I}^2}} [\alpha_n^{\mathrm{I}} l_1 \sin\mathrm{h}(\alpha_n^{\mathrm{I}} l_1) \cos\mathrm{h}(\alpha_n^{\mathrm{I}'} x_1) \\
&\quad - \alpha_n^{\mathrm{I}} x_1 \cos\mathrm{h}(\alpha_n^{\mathrm{I}} l_1) \sin\mathrm{h}(\alpha_n^{\mathrm{I}} x_1)] \sin(\alpha_n^{\mathrm{I}} y_1)
\end{aligned}$$

$$- \sum_{s=1}^{\infty} \frac{B_{s4}^{I}}{\Omega_{s4}^{I} \lambda_s^{I'2}} [\lambda_s^{I'} h_1 \cos h(\lambda_s^{I'} h_1) \sin h(\lambda_s^{I'} y_1)$$

$$- \lambda_s^{I'} y_1 \sin h(\lambda_s^{I'} h_1)\} \cos h(\lambda_s^{I'} y_1)] \cos(\lambda_s^{I'} x_1) \quad (4)$$

where

$$A_0^I, B_0^I, A_{n1}^I, B_{s1}^I, A_{n2}^I, B_{s2}^I, A_{n3}^I, B_{s3}^I, A_{n4}^I, B_{s4}^I$$

are unknown coefficients which are determined from the boundary conditions, and where

$$\alpha_n^I = \frac{n\pi}{h_1}, \quad \alpha_n^{I'} = \frac{(2n-1)\pi}{2h_1}, \quad \lambda_s^I = \frac{s\pi}{l_1}, \quad \lambda_s^{I'} = \frac{(2s-1)\pi}{2l_1}$$

Airy's stress functions χ^i (i = II, III, IV, V) are expressed by changing the right shoulder suffix I shown in Eq. (4) into II, III, IV and V, respectively. In addition, the variables l_1, h_1 and y_1 are changed into l_i, h_i and y_i (i = 2, 3, 4, 5). Substituting Airy's stress functions χ^I, χ^{II}, χ^{III}, χ^{IV} and χ^V into Eq. (1) and (2) for analyzing each finite strip, simultaneous equations are obtained. By solving the simultaneous equations, the unknown coefficients are determined. Using the determined coefficients, each stress (plane stress state) is obtained. The other finite strips are analyzed in the same way.

2.2. Analysis of Laminated Sandwich Beams Subjected to Impact Loads

In order to analyze the impact response of the beams, F.E.M. code DYNA3D is employed. Figure 3 shows an example of element division in impact analysis of three-layered laminated sandwich beams. A solid cylinder, 10 mm in diameter and 20 mm in height, is dropped from the hight, H, to the upper surface of the laminated sandwich beam. The length of the beam is 500 mm.

3. EXPERIMENTS

Figrue 4 shows a schematic of the experimental setup. In the experiments, deflections of the beams subjected to static loads are measured using a displacement transducer. The material of finite strips [I], [III] and [V] is chosen as steel ($E_1 = E_3 = E_5 = 206$ Gpa), finite strips [II] and [IV] as acrylic resin ($E_2 = E_4 = 3.$ 4 GPa). The dimensions are chosen as $2l = 500$ mm and $2h_1 = 2h_2 = 2h_3 = 2h_4 = 2h_5 = 10$ mm. Figure 5 shows an experimental setup for measuring strain response when an impact load is applied to a three-layered laminated sandwich beam. Strains of the beams subjected to impact loads were measured using strain gauges. Strain gauges are attached at the positions $x = 100, 150$ and 200 mm from the center. The dimensions of the finite strips are chosen as $2l = 500$, $2h_1 = 2h_3 = 10$ and $2h_2 = 3$ mm. The material of finite strip [I] and [III] is chosen as steel

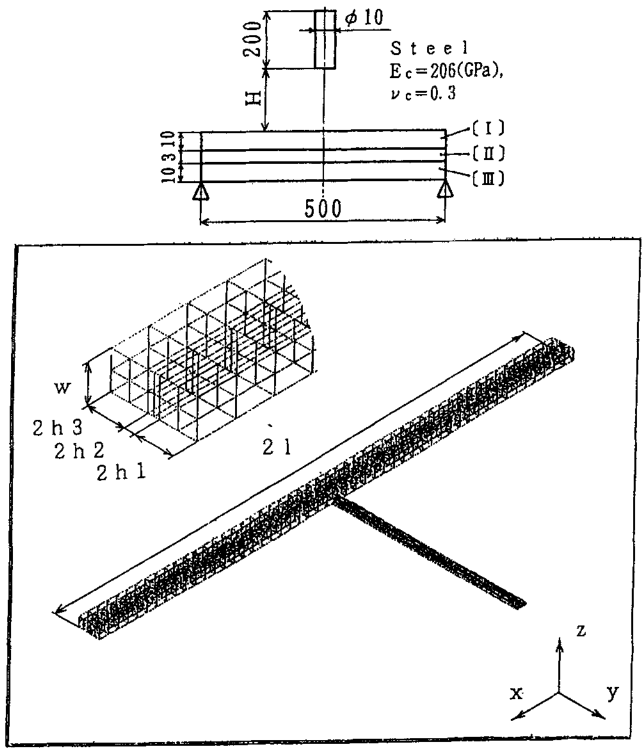

FIGURE 3 An example of element division in impact analysis of three-layered laminated sandwich beams. (F.E.M. code employed is DYNA3D.)

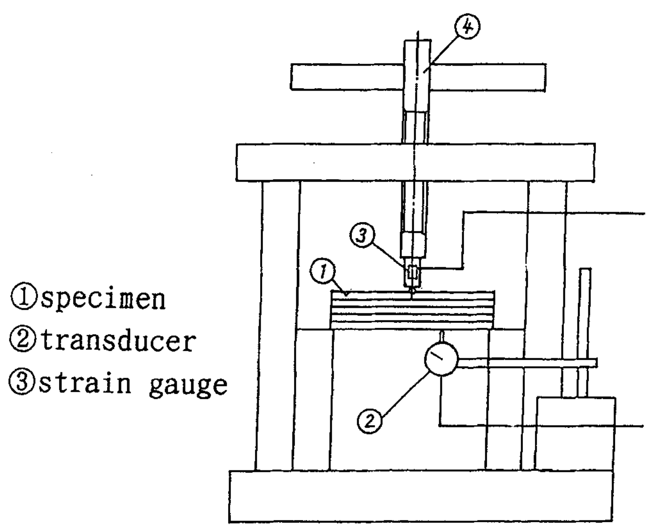

FIGURE 4 A schematic experimental setup (static load).

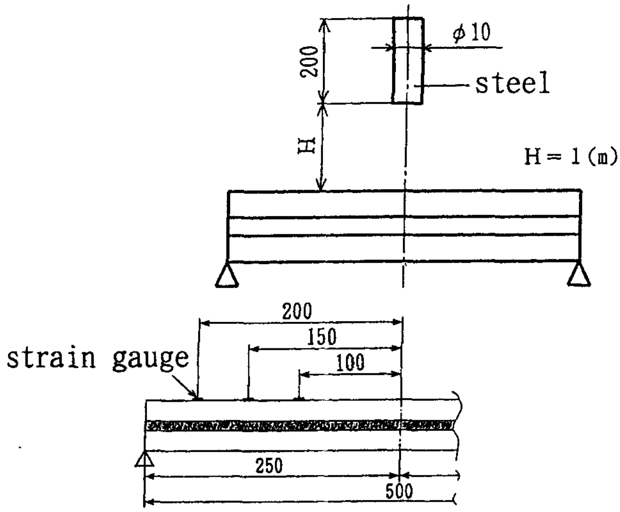

FIGURE 5 A schematic experimental setup for measuring strain response (an impact load is applied to a three-layered laminated sandwich beam).

($E_1 = E_3 = 206$ GPa) and finite strip [II] as epoxy resin ($E_2 = 3.4$ GPa). The height H is chosen as 1 m.

4. ANALYTICAL RESULTS AND COMPARISON WITH EXPERIMENTAL RESULTS

4.1. Analytical Results

Figure 6 shows the stress distributions at the interface ($y_4 = -h_4$) when a laminated sandwich beam is subjected to a static load. In the numerical calculations, Young's moduli are chosen as $E_1 = E_3 = E_5$, $E_2 = E_4$ and Poisson's ratios are chosen as 0.3. The abscissa is the normalized distance x/l and the ordinate indicates the normalized stresses σ_x/σ_{ym}, σ_y/σ_{ym} and τ_{xy}/σ_{ym}, where σ_{ym} is the mean normal stress. Figure 7 shows the effect of Young's moduli among each strip on the interface stress ($y_4 = -h_4$) when a laminated sandwich beam is subjected to a static load. The abscissa is the normalized distance x/l and the ordinate indicates the normalized maximum principal stress σ_1/σ_{ym}. It is found that singular stresses occur at both ends of the interfaces. In addition, it is seen that the singular stress increases with a decrease of the ratio E_1/E_2. Figure 8 shows the effect of the thickness ratio, h_1/h_2, on the normalized maximum principal stress distribution, σ_1/σ_{ym}, at the interface $y_4 = -h_4$. As a result, it is seen that the singular stress increases with an increase of the ratio h_1/h_2.

Figure 9 shows the numerical results of stresses σ_x, σ_y and τ_{xy} at the interface between finite strips [I] and [II] ($x = 250$ mm) when a three-layered laminated sandwich beam is subjected to an impact load. Young's modulus of each layer is

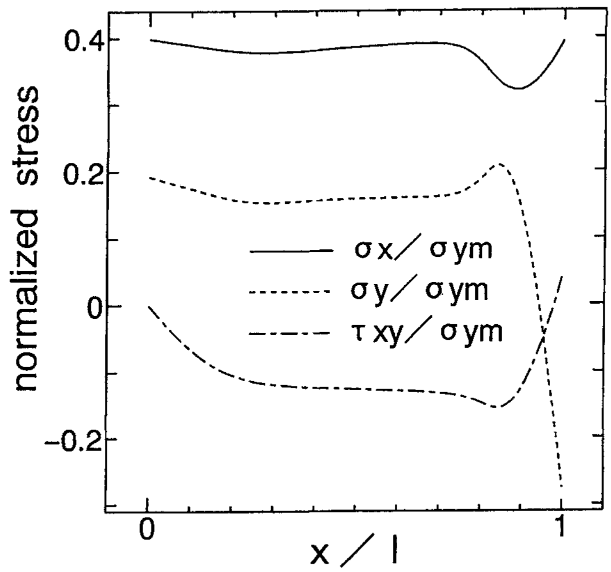

FIGURE 6 Stress distributions at the interface ($y_4 = -h_4$) when a laminated sandwich beam is subjected to a static load.

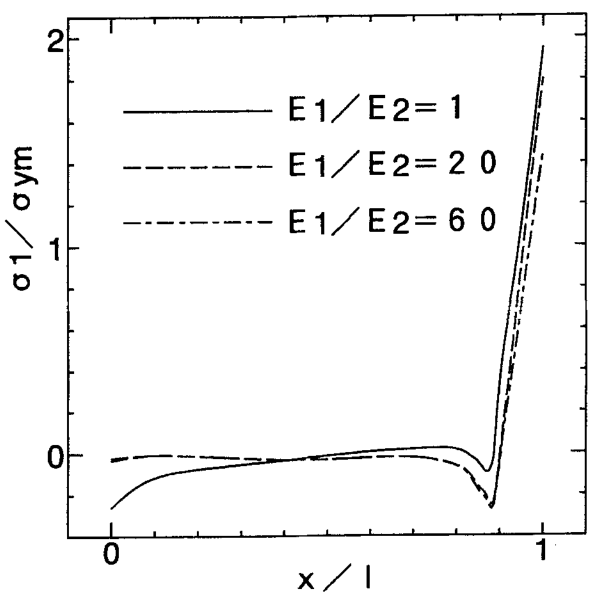

FIGURE 7 Effects of Young's moduli ratio, E_1/E_2, among each strip on the interface stress ($y_4 = -h_4$) when a laminated sandwich beam is subjected to a static load.

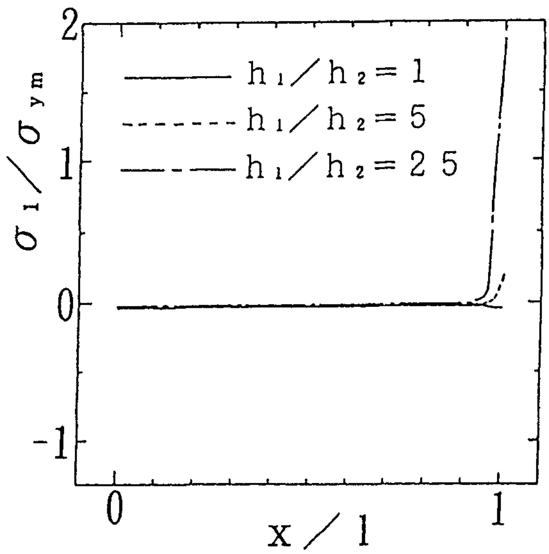

FIGURE 8 Effect of the thickness ratio, h_1/h_2, on the normalized maximum principal stress distribution, σ_1/σ_{ym}, at the interface $y_4 = -h_4$.

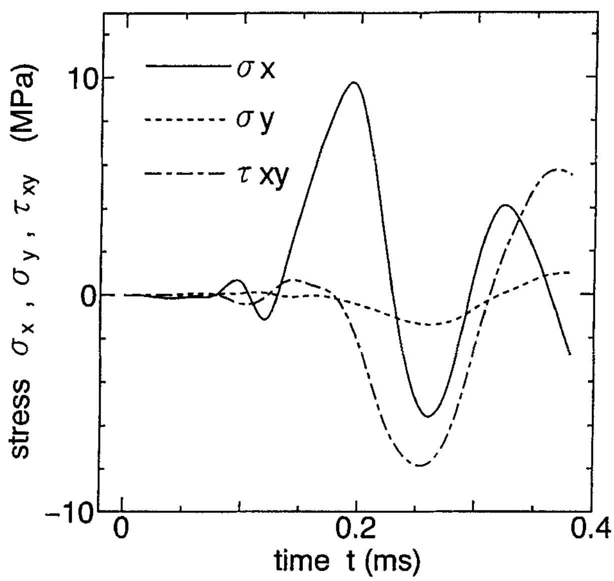

FIGURE 9 Numerical results of stresses σ_x, σ_y and τ_{xy} at the interface between finite strips [I] and [II] ($x = 250$ mm) when a three-layered laminated sandwich beam is subjected to an impact load.

chosen as $E_1 = E_3 = 206$ GPa and $E_2 = 3.4$ GPa. The steel bar is dropped from a height of 1 m. The abscissa is the time from release of the steel bar and the ordinate indicates the stresses σ_x, σ_y and τ_{xy} at the interface between finite strips [I] amd [II]. Figure 10 shows the numerical results when a three-layered laminated sandwich

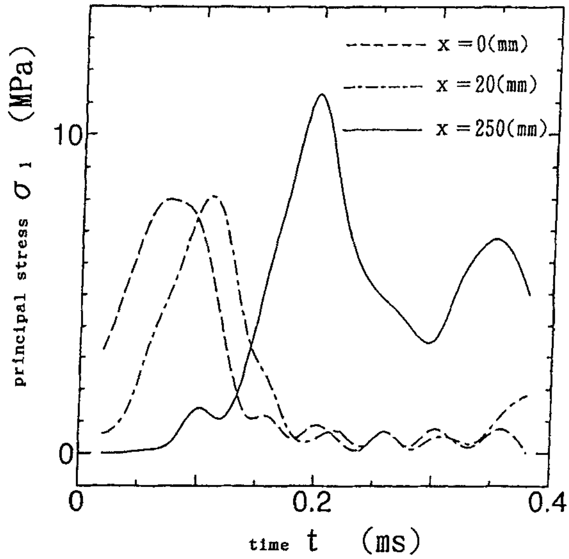

FIGURE 10 Numerical results of the maximum principal stress at the interface between finite strip [I] and [II] when a three-layered laminated sandwich beam is subjected to an impact load.

beam is subjected to an impact load. Young's modulus of each layer is chosen as $E_1 = E_3 = 206\,\text{GPa}$ and $E_2 = 3.4\,\text{GPa}$. The ordinate indicates the maximum principal stresses, σ_1, at the interface between finite strips [I] and [II] ($x = 0, 20$ and $250\,\text{mm}$). From the results, it is seen that the maximum principal stress, σ_1, occurs at the time of 0.2 ms and at the position $x = 250\,\text{mm}$, that is, at the edge of the beam.

Figure 11 shows the stress distribution at the interface between finite strips [I] and [II] (at the time of 0.2 ms). Figure 12 shows the maximum principal stress at the edges of laminated beam. From the results, it is found that the maximum principal stress, σ_1, occurs at the position 2 shown in Figure 12, that is, at the edge of the interface between finite strip [I] and [II].

Figure 13 shows the effects of Young's modulus ratio E_1/E_2 on the maximum principal stress distribution, σ_1, at the interface between finite strip [I] and [II] ($y_1 = -h_1$). At the edge of the interface, the stress σ_1 increases as the ratio E_1/E_2 increases. Figure 14 shows the comparisons of the maximum principal stress distributions between a static load and an impact load. In the case of the static load, the load is applied by placing the bar (10 mm in diameter) at the upper surface of the beam. The difference between the two cases is substantial.

4.2. Comparisons Between Numerical and Experimental Results

Figure 15 shows the comparison between the numerical and the experimental results when a static load is applied. The ordinate is the displacement, v, for the load $p = 980\,\text{N}$. A fairly good agreement is seen between the numerical and the

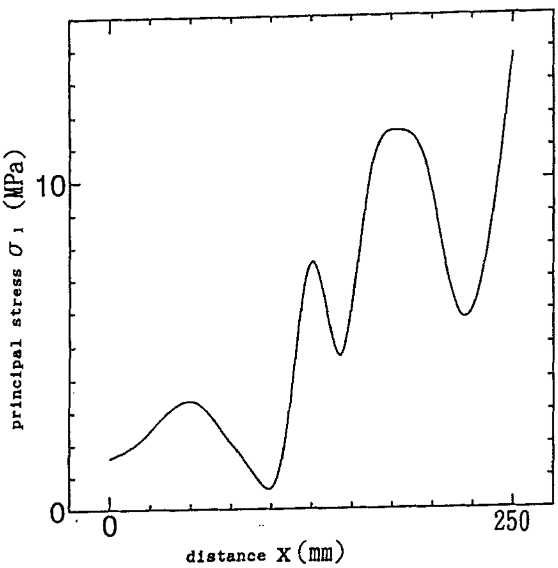

FIGURE 11 Principal stress distribution at the interface between finite strip [I] and [II] when the time is 0.2 ms.

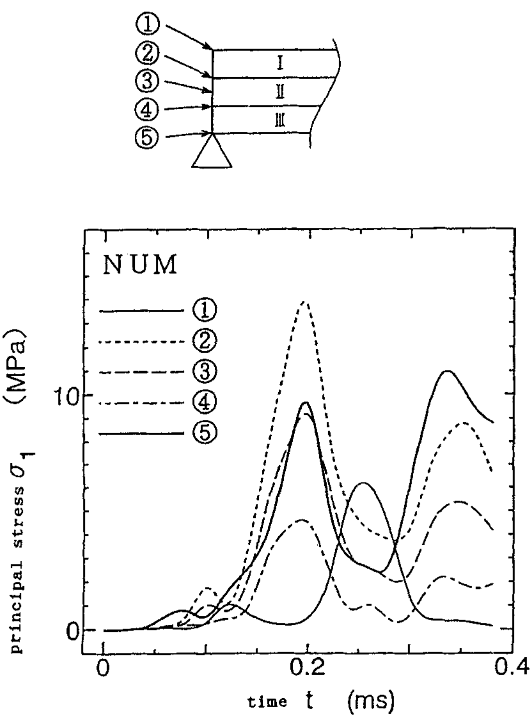

FIGURE 12 Maximum principal stress at the edges of the interfaces.

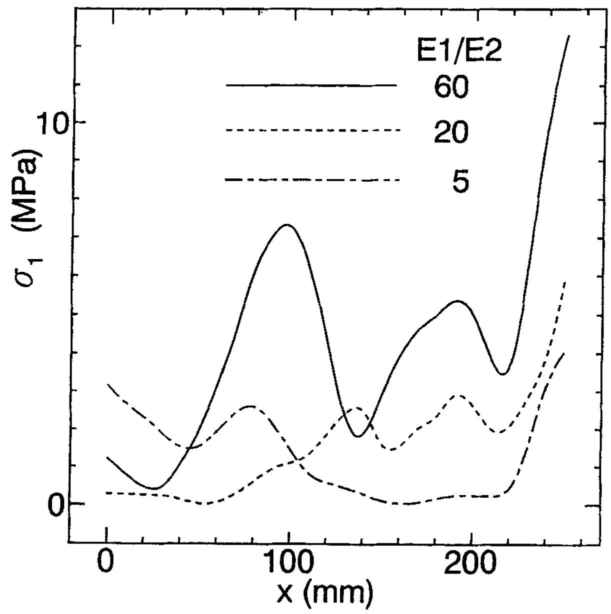

FIGURE 13 The effects of Young's modulus ratio, E_1/E_2.

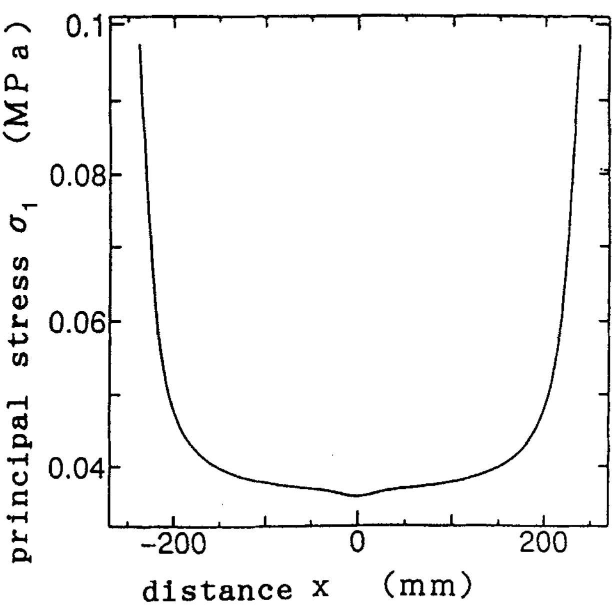

FIGURE 14(a) Interface stress distribution between finite strip [I] and [II] in a static load.

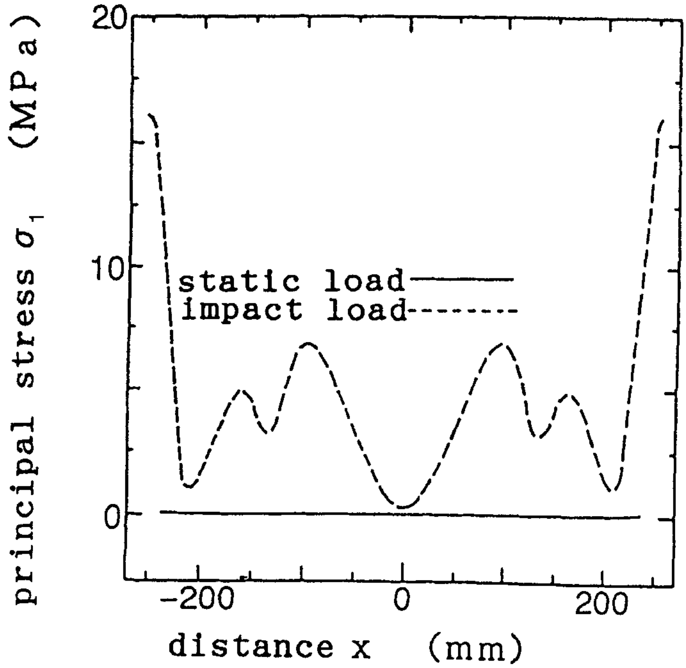

FIGURE 14(b) Comparison between the maximum principal stress distributions in a static load and an impact load.

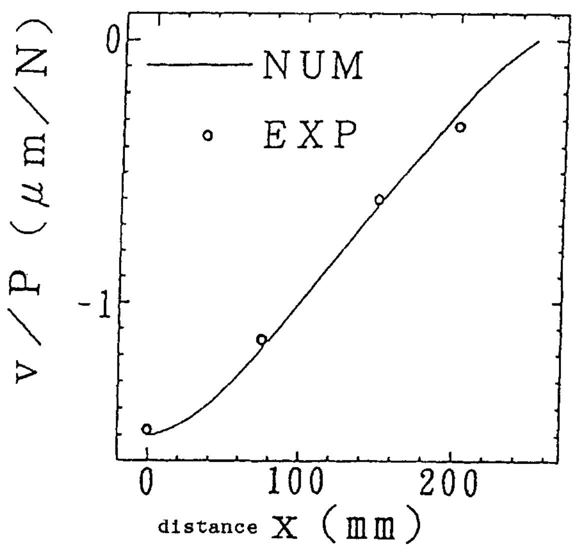

FIGURE 15 Comparison between the numerical and the experimental results when a static load ($P = 980$ N) is applied.

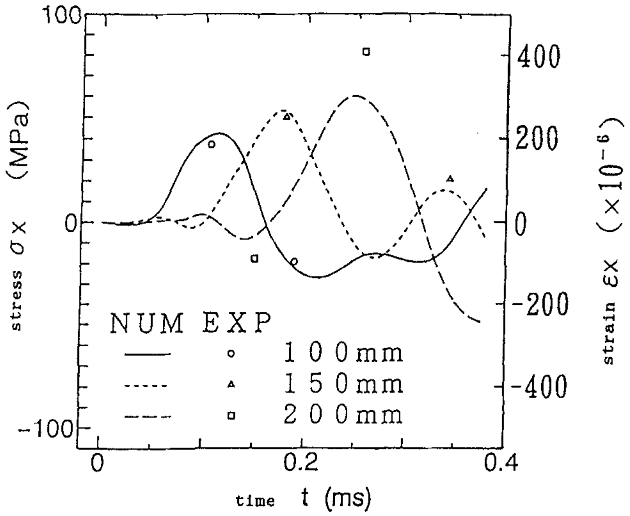

FIGURE 16 Comparison between the numerical and the experimental results when an impact load is applied. (At the positions of $x = 100, 150, 200\,\mathrm{mm}$).

experimental results. Figure 16 shows the comparisons when an impact load is applied. The ordinate indicates the strain, ε_x, at the positions of $x = 100, 150$ and 200 mm. A fairly good agreement is found between the numerical and the experimental results.

5. CONCLUSIONS

This paper has dealt with the stress analysis of laminated sandwich beams subjected to static and impact loads. The results obtained are as follows.

1. The interface stress distribution of five-layered laminated sandwich beams is analyzed as a five-body contact problem by using a two-dimensional theory of elasticity. The effects of the ratios of Young's moduli among each finite strip on the interface stress distribution are clarified. It is found that a singular stress occurs at the edge of the interfaces and it increases with a decrease of the Young's modulus ratio, E_1/E_2.
2. The interface stress response of three-layered laminated sandwich beams is analyzed by using F.E.M. It is found that the singular stress occurs at the edges of the interfaces and that it increases with an increase of the Young's modulus ratio, E_1/E_2.
3. Experiments were conducted on the displacement in static loads. In addition, experiments were carried out on the strains when impact loads were applied to laminated sandwich beams. A fairly good agreement is seen between the numerical and the experimental results.

References

1. K. Kommoch, *Experimental Mechanics*, **20**, 80 (1980).
2. F. C. Moon, *J Composite Materials*, **6**, 66–79 (1972).
3. Y. Xia, C. Ruiz, *J. Impact Engng.*, **7**, 415–428 (1988).
4. Y. Xia, *I. Mech. E.*, Conference Publications, **2**, 87–92 (1989).
5. B. V. Sankar, C. T. Sun, *AIAA J.*, **23**, 1962–1969 (1985).